KDIS Working Paper 22-04

Measuring Interdependence of Inflation Uncertainty

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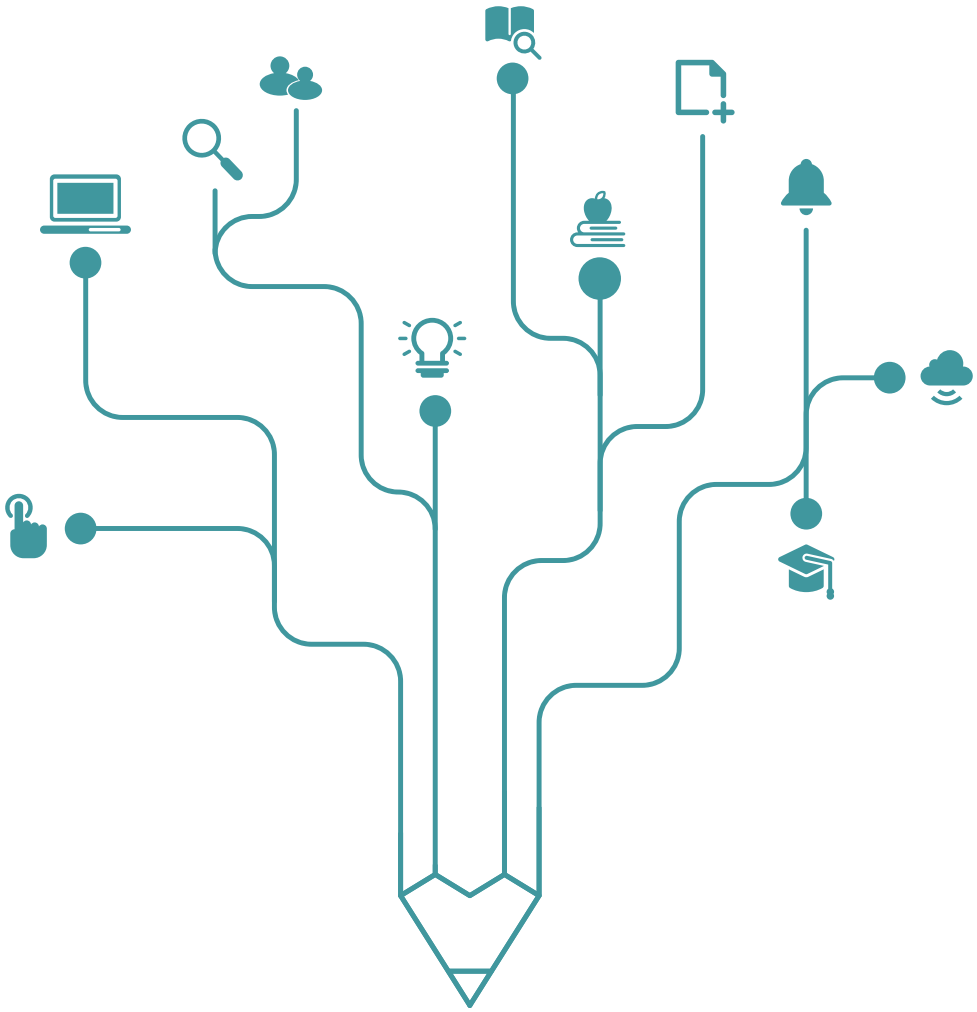

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August 4, 2022

Abstract

The unprecedented fiscal and monetary policy responses during the COVID-19 crisis have increased uncertainty about inflation. During crises periods, the strength of the transmission of inflation uncertainty shocks from one country to another tends to intensify. This paper examines empirical methodologies to measure the strength of the interdependence of inflation uncertainty between the UK and the euro area. We first estimate inflation uncertainty by *ex post* forecast errors from a bivariate VAR GARCH model. The interdependence of uncertainty is estimated using a probability model. The results imply that the spillover of uncertainty is stronger for uncertainty about distant future than near future. The evidence from quantile regressions shows that such empirical method could suffer from bias if endogeneity is not properly addressed. To identify structural parameters in an endogeneity representation of interdependence, we exploit heteroskedasticity in the data across different regimes determined by the ratio of variances. The results no longer exhibit stronger interdependence at longer horizons. Estimated by different sample periods, the strength of the propagation of inflation uncertainty intensifies during the Global Financial Crisis while the interdependence significantly weakens during the post-crisis period.

JEL Classification: E37, E52, F42

Keywords: inflation uncertainty, interdependence, GARCH, copulas, at-risk, conditional forecasting, identification through heteroskedasticity.

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I Introduction

The unprecedented fiscal and monetary policy responses to address the COVID-19 pandemic has prompted recent debates about the inflation prospects after the decades of low and stable inflation (e.g. Ball et al., 2021; Blanchard, 2021). The studies based on survey conducted in the early stages of the pandemic also suggest that uncertainty about inflation has rapidly risen since the pandemic (Armantier et al., 2021, Coibion et al., 2020). The heightened inflation uncertainty, along with the actual increases in inflation, has been seen globally and it is argued that multiple driving factors are relevant–including the uncertain path of the pandemic and economic recovery as well as the surge in commodity prices and its volatility followed by the pandemic-induced supply disruptions.

While it is well studied that inflation co-moves closely across countries (e.g. Monacelli and Sala, 2009; Ciccarelli and Mojon, 2010; Mumtaz and Surico, 2012; Bäurle et al., 2021), the research on the interdependence of inflation uncertainty are not well established. A large number of studies have analyzed the potential channels of the co-movement in inflation: common macroeconomic shocks (e.g. global commodity prices), trade, labor market channel through migration, exchange rate regimes, and financial integration.[1] Through similar mechanisms, uncertainty about inflation of a country can be transmitted to other countries and the transmission of inflation uncertainty shocks may exhibit different patterns over the various stages of global macro-financial cycles. The objective of this paper is to explore the various empirical methodologies to measure the strength of the interdependence of inflation uncertainty. In particular, this paper showcase the empirical framework by examining the case of the euro area and the United Kingdom (UK) with close economic, trade and financial integration.

This study extends a probabilistic model to characterize the entire distribution of inflation uncertainty in a two-country setting.[2] We first estimate inflation uncertainty by *ex post* forecast errors

[1] See, for example, Henriksen et al. (2013) (common macroeconomic shocks), Melitz and Ottaviano (2008) (trade openness), Bentolila et al. (2008) (migration channel), Calvo and Reinhart (2002) (foreign exchange regimes) and Rey (2016) (financial integration).

[2] Broadly speaking, the probabilistic model can also be referred as the 'at risk' approach following Adrian et al. (2019). While the empirical framework and estimation procedures of this paper differ substantially from the recent 'at risk' literature, the idea of estimating entire conditional distributions assuming a specific parametric distribution is in line with the one in Adrian et al. (2019) and the following literature.

from a two-country bivariate VAR GARCH model. By defining the uncertainty measure as forecasts errors rescaled by their unconditional and conditional variance-covariance matrix, we allow the measure of inflation uncertainty to distinguish upside and downside surprises in inflation.

Next a two-step procedure for a probabilistic model is applied to evaluate the interdependence of inflation uncertainty between the UK and the euro area. The first step is to estimate the best-fit marginal density against two non-Gaussian distributions that could potentially account for heavy tail and skewness behavior of inflation uncertainty. As for the candidates, two piece normal distribution (TPN) and weighted skewed normal distribution (WSN) are considered. The use of TPN (Wallis, 2004) follows from the convention of central banks' fan chart to evaluate the balance of uncertainty. The alternative density, WSN (Makarova, 2018), is a synthetic distribution which is designed to reveal monetary policy responses to upside or downside inflation uncertainty. The results from fitting uncertainty to two different marginal distribution TPN suggest the possibility of long right tails in the distributions of the euro area inflation uncertainty. The results from WSN suggest monetary policy reactions by the ECB tend to be stronger to upside inflation surprises than downside surprises regardless of forecasting horizons. As oppose to the ECB's case, the BOE's monetary policy stance tends to be dovish to long term inflation surprises. But, for short term inflation surprises, the BOE's monetary policy responses are likely to be hawkish, as seen in the ECB's case.

The second step of the procedure is to estimate the conditional probability distribution of inflation uncertainty of two economies using copulas. Copulas is a flexible tool to construct a multivariate distribution by combining marginal densities estimated separately from the conditional probability distribution (see, e.g., Rodriguez, 2007; Smith and Vahey, 2016). Among various bivariate copulas, Frank copula is chosen to allow for asymmetric dependence structures without favoring either upper or lower tail dependence. The interdependence of inflation uncertainty of two economies can be summarized in the estimated copula parameters and the results unanimously point to higher dependence for the uncertainty about distant future than near future.

While the probabilistic model provides succinct estimates to measure interdependence, it could

suffer from endogeneity problems (Rigobon, 2019). If there exists heteroskedasticity in the error terms of the structural model, the conditional density estimated by a probabilistic model can be biased. One typical example applicable to our empirical framework would be the case where the variance of inflation uncertainty shocks increases during the crisis period. To detect whether the estimates from the probabilistic model suffer from potential endogeneity bias, we run quantile regressions (Koenker and Bassett, 1978). The evidence from quantile regressions suggests potential biases in the copula estimates and the conditional distribution derived from the copula.

To identify structural parameters in an endogeneity representation of interdependence, we exploit heteroskedasticity in the data across different regimes determined by the ratio of variances as proposed in Rigobon (2019). The estimation is based on minimum distance criteria and statistical significance is bootstrapped. The results no longer exhibit stronger interdependence at longer horizons across all regimes. Instead, estimated by different sample periods, the strength of the propagation of inflation uncertainty intensifies during the Global Financial Crisis (GFS) while the interdependence is significantly muted during the post-crisis period.

Our paper contributes to a large literature on the measures of inflation uncertainty. Following Ball (1992), a long-standing practice in the literature is to examine the relationship between the level of inflation and inflation uncertainty measured by its volatility (see, for example, Grier and Perry, 2000; Kontonikas, 2004). Among many recent empirical studies, Caporale et al. (2012) examine the relationship between inflation uncertainty and inflation level in European countries by employing GARCH-type models. Survey-based disagreement measures of inflation have also been widely studied (see, e.g., Holland, 1995; Giordani and Söderlind, 2003; Clements and Harvey, 2011; Wright, 2011). Binder (2017) constructs micro-level inflation uncertainty measures by quantifying the uncertainty associated with round number responses in the survey data. Our approach departs from the existing literature by constructing an *ex post* unpredictability measure of inflation uncertainty that is close to the definition in Friedman (1977). Uncertainty, defined as the component that had not been predictable at the time of forecasting, is computed by pseudo out-of-sample forecasting errors as in Stock and Watson (2007). This framework provides a parsi-

monious way to construct a measure for inflation uncertainty from a multivariate GARCH model of two economies.

This paper is also related to the burgeoning literature on the 'at risk' approach which aims at estimating and evaluating conditional distributions of economic variables (see Adrian et al., 2019; López-Salido and Loria, 2020; Sokol, 2021, among many). Our approach differs from the existing ones in that it considers a two-country model rather than a variable conditional on other macroeconomic or financial variables to assess entire conditional distributions.

From an econometrics point of view in estimating the strength of interdependence, this paper is closely related to Pesaran and Pick (2007). They define contagion as a situation whereby a crisis in one country increases the probability of a crisis in another country over and above what would be implied by the interdependence in non-crisis periods. By illustrating a two-country framework, they show that the correlation-based tests of contagion could be biased due to endogeneity. Our empirical framework to identify structural parameters of a two-country setup in the presence of heteroskedasticity shares the similar idea of possible endogeneity issues. But we differ from their approach in that we exploit a broader definition of interdependence, that encompasses crises and non-crises periods alike, for identification through heteroskedasticity.

The rest of the paper is organized as follows. Section II explains inflation uncertainty computed by forecast errors from a bivariate VAR GARCH model. Section III estimates the interdependence of inflation uncertainty by a probability model. In Section IV, an endogenous model of interdependence is introduced to discuss potential biases of a probability model and the interdependence of inflation uncertainty is estimated by identification through heteroskedasticity. Section V concludes.

II Estimating inflation uncertainty

Inflation uncertainty is measured by *ex post* forecast errors from a bivariate VAR BEKK GARCH (1,1) model using inflation rates of the UK and the euro area. The main advantage of the forecasting model is that it is parsimonious while taking account for potential interdependence of two

economies and time-varying volatility in inflation. Inflation uncertainty, denoted by $U_{t,h}$, is defined as below.

$$U_{t,h} = \Sigma_{t,h}^{1/2} \Sigma_{t|t-h}^{-1/2} e_{t|t-h} = \Sigma_{t,h}^{1/2} \Sigma_{t|t-h}^{-1/2} (\pi_t - \pi_{t|t-h}) \tag{1}$$

where $e_{t|t-h}$ is the forecast error conditional on the information available at the period of the prediction, i.e. the h-period ahead forecasts is made at time $t-h$. $\Sigma_{t,h}$ denotes the unconditional variance-covariance matrix of e_t and $\Sigma_{t|t-h}$ the conditional variance-covariance matrix of $e_{t|t-h}$. The forecast errors are computed by the differences between the realized inflation at t (π_t) and the h-step ahead forecast of inflation at t ($\pi_{t|t-h}$). To standardize the measure, the forecast errors are multiplied by the square root of the unconditional variance-covariance matrix and divided by the square root of the conditional variance-covariance matrix.

The monthly data for inflation is retrieved from the Eurostat database and the sample period is from January 1997 to March 2016. For both countries, inflation series are found to be I(1) and the first differences of the raw inflation series are used for the maximum likelihood estimation. Autoregressive order of VAR model is determined by Ljung-Box autocorrelation test for residuals. The minimal number of lags is chosen to ensure the residuals exhibit no autocorrelation at 5 percent significance level.

Based on the estimated VAR BEKK GARCH (1,1) model, the h-step ahead forecasts for $h = 1, 2, \ldots, 24$ months are estimated recursively with the initial recursion using the first 80 observations.[3] The resulting conditional and unconditional variance-covariance matrices are also obtained recursively. The h-step forecasts by maximum likelihood estimation can suffer from spurious dependence when $h > 1$. In order to tackle this issue, Vector Moving Average (VMA) decomposition is used for the estimation of the Mean Squared Error (MSE) matrix of the forecasts (see Lütkepohl, 2005).

Figure 1 plots inflation uncertainty of the UK and the euro area for the selected forecast horizons ($h = 3, 6, 12, 18, 24$). Inflation uncertainty rose rapidly after the Great Financial Crisis in

[3]The forecast yields 151 ($= 231 - 80$) *one-step-ahead* forecast errors, 150 *two-step-ahead* forecast errors, ... up to 128 *24-step-ahead* forecast errors.

2008 for both countries, followed by a significant decline below the average level. Positive values of inflation uncertainty imply that the realization of inflation had not been predicted at the time of forecasting and the unanticipated elements caused inflation to move upwardly. Similarly, negative values of inflation uncertainty measure indicate that the realized inflation rates were lower than the prediction by the two-country VAR-GARCH model. The descriptive statistics of inflation uncertainty (Table 1) suggest that inflation uncertainty may be better characterized with non-Gaussian density functions with non-zero skewness and/or heavy tails.

Correlation coefficients can be considered as a simple measure for interdependence of inflation uncertainty between the two economies. The Pearson's correlation coefficient captures only linear correlation and thereby is considered to be insufficient for a measure of dependence in the case of heavy tail or asymmetric dependence (Cont, 2001; Boyer et al., 1997). Therefore, rank correlation coefficients are computed in Figure 2. The average Spearman's correlation coefficient is 0.29 while Kendall's correlation coefficient is 0.21. The uncertainty measures with longer forecast horizons tend to exhibit higher correlations.

III Measuring interdependence by a probability model

This section illustrates a probability model to measure interdependence of inflation uncertainty of two economies. Probability models assume that a change in the probability of the two events occurring together reflects the strength of the dependence of those two events. In this paper, copulas, a tool widely used in finance for modeling extreme events, are applied to measure interdependence for non-Gaussian marginal densities. The main advantage of copulas is its flexibility in combining different parametric family univariate distributions. Also, the choice of a dependence model, a copula function, can be independent from the choice of the marginals. Following the Inference Function for Margins (IFM) method, as in Joe and Xu (1996), two steps of estimation procedures are sketched in this section. First, univariate marginal distributions are estimated by the simulated minimum distance criteria. Given the best-fit univariate distributions, a copula parameter is estimated by the maximum likelihood estimation.

A Marginal density function

Inflation uncertainty is estimated against two non-Gaussian univariate distributions from the same distribution family of skew normal distribution: Two Piece Normal (TPN; Wallis, 2004) and Weighted Skewed Normal (WSN; Makarova, 2018).

The choice of TPN density follows from the convention of central banks' fan chart. Starting from the Bank of England, fan chart is well-known and most widely used presentation of the probabilistic forecasts of inflation.[4] Fan chart considers both the degree of uncertainty and the balance of uncertainty around the forecast is assessed using TPN distribution (Britton et al., 1998). The *pdf* of TPN distribution is defined as follows (Wallis, 2004).

$$f_{TPN}(x;\sigma_1,\sigma_2,\mu) = \begin{cases} A exp\{-(x-\mu)^2/2\sigma_1^2\} & \text{if } x \leq \mu \\ A exp\{-(x-\mu)^2/2\sigma_2^2\} & \text{if } x > \mu \end{cases} \tag{2}$$

where $A = (\sqrt{2\pi}(\sigma_1+\sigma_2)/2)^{-1}$. If $\sigma_1 = \sigma_2$, it collapses to a Normal distribution. If $\sigma_1 < \sigma_2$, the distribution is positively skewed (long right tail).

The other candidate for univariate density is WSN. Derived from a combination of two normal distributions, WSN is a customised skew normal distribution which aims at decomposing uncertainty into epistemic and ontological components. Ontological uncertainty is assumed to be complete randomness formed by public knowledge whereas epistemic uncertainty indicates the uncertainty based on expert knowledge (Walker et al., 2003). To illustrate the distribution, we denote the inflation uncertainty measured by forecast errors as U, omitting the subscripts, t, h, for simplicity. It is assumed that U is decomposed by two components – the baseline forecast error (X) and the signal part based on the revised forecast error from expert knowledge (Y).

$$U = \underbrace{X}_{\text{baseline forecast error}} + \underbrace{\alpha \cdot Y \cdot I_{Y>m} + \beta \cdot Y \cdot I_{Y<k}}_{\text{Signal part based on revised forecast error}} \tag{3}$$

[4] A recent study using the TPN in examining price is Sokol (2021).

where $I_{Y>m}$ is an indication function that gives 1 if revised forecast errors are larger than a certain threshold, $m \geq 0$. Similarly, $I_{Y<k}$ is an indication function that gives 1 if revised forecast errors are smaller than a certain threshold, $k \leq 0$. Hence, the signal part will be switched on when either (i) $Y > m \geq 0$ or (ii) $Y < k \leq 0$ holds. X and Y are bivariate Normal distributions with mean zero, constant and identical variances (σ^2), and correlation coefficient, ρ. This implies that if $\alpha = \beta = 0$, WSN reduces to a Normal distribution.

The key assumption of the WSN distribution is that the *ex post* inflation uncertainty can be decomposed into the baseline forecast errors from public knowledge and the revised forecast errors based on expert knowledge given expected central bank's monetary policy decisions. Assume that the baseline forecast error (X) is initially established. Further assume that the forecast error based on expert knowledge is positive and larger than a certain threshold ($Y > m \geq 0$) and experts would know that a central bank with expert knowledge is expected to respond to this upside inflation surprise with hawkish policy actions, for example, by increasing the policy rate. Then the *realized* forecast error would be revised downwards ($\alpha Y < 0$ with $\alpha < 0$) from the initial baseline forecast errors. The magnitude of the effect of monetary policy tightening on inflation can be summarized in the parameter, α. Similarly, in the opposite case where the forecast error based on expert knowledge is negative and smaller than a certain threshold ($Y < k \leq 0$), β is assumed to be negative and depicts the magnitude of the effect of monetary policy easing to downside surprises ($\beta Y > 0$ with $\beta < 0$). The comparison between α and β in absolute value provides interesting intuition. If $|\alpha|$ is greater than $|\beta|$, it implies that the central bank tends to react more aggressively towards upside uncertainty than downside uncertainty.

The *pdf* of WSN distribution is as follows (Charemza et al., 2015).

$$
\begin{aligned}
f_{WSN}(x;\alpha,\beta,m,k,\rho) = & \frac{1}{\sqrt{A_\alpha}}\phi\left(\frac{x}{\sqrt{A_\alpha}}\right)\Phi\left(\frac{B_\alpha x - mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}}\right) \\
& + \frac{1}{\sqrt{A_\beta}}\phi\left(\frac{x}{\sqrt{A_\beta}}\right)\Phi\left(\frac{-B_\beta x + kA_\beta}{\sqrt{A_\beta(1-\rho^2)}}\right) \\
& + \phi(x)\cdot\left[\Phi\left(\frac{m-\rho x}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{k-\rho x}{\sqrt{1-\rho^2}}\right)\right] \qquad (4)
\end{aligned}
$$

where ϕ and Φ is the *pdf* and *cdf* of a standard normal distribution, respectively. $A_\tau = 1+2\tau\rho+\tau^2$ and $B_\tau = \tau + \rho$ for $\tau = \alpha, \beta$.

To find the best-fit marginal density for inflation uncertainty, the simulated minimum distance estimators method (SMDE) is applied.[5] The empirical histograms of inflation uncertainty estimated in Section I are fitted to the simulated density functions using a minimum distance criterion.

$$\hat{\theta}_{\text{SMDE}} = \arg \min_{\theta} \left[\xi \left(HD(d_n, f_{r,\theta}) \right)_{r=1}^{R} \right] \tag{5}$$

where d_n is the empirical histogram from the original data, $f_{r,\theta}$ is the simulated Monte Carlo approximation of theoretical densities with total R replications. ***HD*** is Hellinger distance measure[6] and ξ denotes the aggregating operator.

Since the number of parameters to be estimated in WSN $(\alpha, \beta, m, k, \rho)$ is larger than that of TPN $(\sigma_1, \sigma_2, \mu)$, it is necessary to impose restrictions on WSN parameters. Only α, β, σ in WSN are estimated by imposing restrictions on m, k, and ρ. It is assumed that $m = -k = \sigma$. In terms of the restriction on ρ, we consider two cases: constant ρ $(= 0.75)$ and ρ decaying exponentially from 0.75 to 0.25 as forecast horizon increases.[7] The second case is to reflect the tendency that the covariance between public and expert knowledge decreases along with the forecast horizons.

Table 2 shows the results of the estimation of two marginal distributions with the selected horizons $(h = 6, 12, 18, 24)$ and under the assumption of exponentially decreasing ρ.[8] First, the estimated WSN parameters, α and β, can be considered as the experts' adjusters based on their expectation on monetary policy reactions to inflation surprises. In the UK case, the absolute value of α is greater than the absolute value of β for shorter horizons $(h = 6, 12)$, implying the prevalence of stronger policy reactions to upside inflation surprises than downside surprises. For longer term horizons $(h = 18, 24)$, the results indicate relatively stronger dovish monetary policy responses

[5]The estimation of skewed normal distributions by the maximum likelihood is known to be inefficient and numerically very complex (see, for example, Azzalini and Capitanio, 1999; Sartori, 2006; Franceschini and Loperfido, 2014).

[6]See Basu et al. (2002) for the definition of Hellinger distance measure.

[7]In particular, the computation is based on $\rho_h = 0.25 + \exp\left[\ln\left(0.75 - 0.25\right) \cdot h\right]$ where $h = 1, 2, \ldots, 24$.

[8]The results of other forecast horizons and different restrictions on ρ are available upon request.

to downside surprises. For the euro area, the estimation results show $|\alpha| > |\beta|$ for all forecast horizons. This suggests that the monetary reactions of the ECB tend to be stronger in response to upside inflation uncertainty than downside, namely a hawkish stance across all forecast horizons.

For the estimated parameters of TPN, the UK results show either $\sigma_1 < \sigma_2$ or $\sigma_1 > \sigma_2$ depending on the forecast horizons without any systematic trend. It is noticeable that σ_1 is much larger than σ_2 for $h = 18$, implying the long left tail. For the euro area, σ_1 is smaller than σ_2 for all horizons, indicating positively skewed (or long right tail) TPN distribution.

In order to select the best-fit marginal distributions, we first examine minimum distance statistics. For the UK case, WSN has smaller MD than TPN in the shorter horizons ($h = 6, 12$) while TPN is preferable for the longer horizons. For the euro area, WSN is selected for most horizons with an exception of the case of $h = 24$.

Next, the probability integral transforms (*pit's*) are computed to examine the goodness-of-fit with the estimated parameters of each marginal density. The *pit's* are the probability of observing the values of a random variable being not greater than its realization values. If the observed density is close to the true but unknown density, *pit's* would be uniform on the interval [0,1]. Figure 3 is the box plot of *pit's* for both WSN and TPN. Both the UK and the euro area inflation uncertainty are fitted better by WSN density across most horizons than by TPN.

To check the compatibility of the data with the uniform distribution, we conduct a simple goodness-of-fit test (the Cramér-von Mises test) using empirical *cdf*. Table 3 presents the test statistics for the selected forecast horizons ($h = 6, 12, 18, 24$).[9] The results support the robustness of parametric estimation. For both WSN and TPN at all horizons, the null hypothesis of uniformity cannot be rejected. By comparing the test statistics, it is also confirmed that WSN is a better fit over TPN for both economies at most of the forecast horizons.

To sum up, the empirical results, including minimum distance statistics, graphical diagnostics of *pit's*, and goodness-of-fit tests, support the choice of WSN against TPN for both the UK and the euro area.

[9]The complete results are available upon request.

B Conditional density function

Given the estimated parameters in the univariate densities, the copula parameter can be estimated by the maximum likelihood estimation. First, denote inflation uncertainty for the UK and the euro area as U_1 and U_2, respectively. The subscript t and h are omitted for simplicity. Consider continuous bivariate joint cumulative density function (*cdf*) of inflation uncertainties, $F(U_1, U_2)$. The univariate marginals for each inflation uncertainty are denoted as $F_1(U_1)$ and $F_2(U_2)$ with inverse quantile functions, ${F_1}^{-1}$ and ${F_2}^{-1}$. Applying the proposition of probability integral and quantile transformation, the joint *cdf* can be written as follows.[10]

$$\begin{aligned} F(U_1, U_2) &= F({F_1}^{-1}(y_1), {F_2}^{-1}(y_2)) \\ &= \Pr[Y_1 \leq y_1, Y_2 \leq y_2] \\ &= C(y_1, y_2) \end{aligned} \tag{6}$$

where $y_1 = F_1(U_1)$, $y_2 = F_2(U_2)$ with a uniform distribution, $\mathcal{U}(0,1)$.[11] C$(\cdot)$ is a copula function that maps the two-dimension support $[0,1]^2$ into the unit interval [0,1].[12] Rewriting the joint *cdf* of inflation uncertainty to obtain the resulting joint *pdf*,

$$F(U_1, U_2) = C(F_1(U_1), F_2(U_2)) \tag{7}$$

[10] Let X be a random variable with density F_X. Let F_X^{-1} be the inverse quantile function of F_X:

$$F_X^{-1}(\alpha) = \inf\{x | F_X(x) \geq \alpha\}$$

$\alpha \in (0,1)$. Then,

(1) If F_X is continuous, the random variable Y, defined as $F_X(X)$, has a uniform distribution. ($F_X(X) \sim \mathcal{U}(0,1)$).

(2) For any uniform distribution $Y \sim \mathcal{U}(0,1)$, we have $F_X^{-1}(Y) \sim F_X$.

[11] This implies $U_1 = {F_1}^{-1}(y_1) \sim F_1$, $U_2 = {F_2}^{-1}(y_2) \sim F_2$.

[12] An *m*-dimensional copula is a function C$(\cdot)$: $[0,1]^m \rightarrow$ [0,1] which satisfies the following conditions:

(1) $C(1, \ldots, 1, a_n, 1, \ldots, 1) = a_n$ for every $n \leq m$;

(2) $C(a_1, \ldots, a_m) = 0$ if $a_n = 0$ for any $n \leq m$;

(3) C is *m*-increasing.

then the joint density (*pdf*) of F is given by the following equation.

$$f(U_1, U_2) = c(F_1(U_1), F_2(U_2)) \cdot f_1(U_1) \cdot f_2(U_2) \tag{8}$$

where c is the density of the copula, partial derivative of $C(\cdot)$ with respect to y_1, y_2. Denote $\theta = (\theta_1, \theta_2, \alpha)$ be all the parameters of F_1, F_2 and C, respectively. Let $U = \{(U_{1t}, U_{2t})\}_{t=1}^{T}$ denote a sample. The log likelihood function can be written as follows.

$$l(\theta) = \sum_{t=1}^{T} \ln\left(c(F_1(U_1;\theta_1), F_2(U_2;\theta_2);\alpha)\right) + \sum_{t=1}^{T} \Big[\ln\left(f_1(U_{1t};\theta_1)\right) + \ln\left(f_2(U_{2t};\theta_2)\right) \Big] \tag{9}$$

Then the maximum likelihood estimator is

$$\hat{\theta}_{\text{MLE}} = \arg\ \max_{\theta}\ l(U_{1t}, U_{2t};\theta) \tag{10}$$

In theory, the copula parameters can be estimated simultaneously with the parameters in marginal distribution by the maximum likelihood estimation. However, in multi-dimension cases, this might lead to high complexity in computation. Hence, the two-step estimation method or the Inference Function for Margins (IFM) method by Joe and Xu (1996) is applied.[13] With the estimated univariate marginal distributions, the copula parameter, γ, is estimated.

$$\hat{\theta}_1 = \arg\ \max_{\theta_1} \sum_{t=1}^{T} \ln\left(f_1(U_{1t};\theta_1)\right) \tag{11}$$

$$\hat{\theta}_2 = \arg\ \max_{\theta_2} \sum_{t=1}^{T} \ln\left(f_2(U_{2t};\theta_2)\right) \tag{12}$$

$$\hat{\gamma} = \arg\ \max_{\gamma} \sum_{t=1}^{T} \ln\left(c(F_1(U_1;\hat{\theta}_1), F_2(U_2;\hat{\theta}_2);\gamma)\right) \tag{13}$$

Among various bivariate parametric families of copulas, Frank copula is chosen. Frank copula

[13]The IFM estimator obtained by the two-step estimation, $\theta_{IFM} := (\hat{\theta}_1, \hat{\theta}_2, \hat{\gamma})$, is known to have a Normal distribution asymptotically (Joe and Xu, 1996).

is a symmetric Archimedean copula[14] and its *cdf* is given by

$$C(y_1, y_2; \gamma) = -\frac{1}{\gamma} \ln \left(1 + \frac{(e^{-\gamma y_1} - 1)(e^{-\gamma y_2} - 1)}{e^{-\gamma} - 1} \right) \tag{14}$$

where $\gamma \in (-\infty, +\infty)$. If $\gamma = 0$, the copula is independent.

γ is estimated positive if heightened uncertainty of one country is associated with higher uncertainty of the other. The copula parameters are estimated by either (i) plugging the marginals of the same forecast horizons or (ii) plugging the marginals that gives highest rank correlation. The dependence structure of the estimated copula can also be summarized by the rank correlations: Kendall's tau (τ) and Spearman's rho (ρ).[15]

Table 4 and Figure 4 shows the estimated γ parameter and the rank correlation coefficients using the marginals of the same forecasting horizons. The copula parameters for all horizons are estimated to be positive and mostly statistically significant. The estimated γ decreases at first and bounces back at around $h = 6$ before decreasing afterwards. However, for the horizons larger than $h = 12$, the estimated γ monotonically increases. The uniformity of the estimated joint distribution is confirmed by the Crémer-von Mises test. The rank correlation coefficients, τ and ρ, exhibit the same trend as the copula parameter. The results imply that the inflation uncertainty of the UK and the euro area contemporaneously affect one another and such spillover effect is stronger for uncertainty about the distant future than the near future.

Table 5 shows the estimation results of the forecast horizons of the euro area inflation uncertainty that have the highest rank correlation coefficients with given horizons of the UK inflation uncertainty. In terms of the Kendall's τ, the short term UK inflation uncertainty series (with fore-

[14] A copula C is Archimedean if there exists a convex, decreasing function $\varphi(\cdot) : (0, 1] \to [0, \infty)$ such that

$$C(y_1, y_2) = \varphi^{-1}(\varphi(y_1) + \varphi(y_2))$$

where $\varphi(\cdot)$ is copula generator and $\varphi(1) = 0$. The examples of Archimedean copulas are Gumbel, Frank, and Clayton. Frank copula is a symmetric Archimedean copula while other two are asymmetric Archimedean copulas. Gumbel copula exhibits greater dependence in the positive tail than in the negative tail while Clayton exhibits greater dependence in the negative tail than in the positive tail. Frank copula is chosen because it can identify the asymmetric dependence structure without favouring either upper or lower tail dependence.

[15] See Appendix A for the details of Frank copula.

cast horizons less than one year) have high correlation with the euro inflation uncertainty series of the forecast horizons from 12 to 14. One-year ahead inflation uncertainty in the UK has the highest correlation with approximately $1\frac{1}{2}$ year ahead uncertainty in the euro area. 16-months to 2-years ahead UK inflation uncertainty series have the highest correlation if paired with 23- to 24-months ahead uncertainty series of the euro area. Spearman's ρ criterion yields fairly similar results to the Kendall's τ criterion with a few exceptions in the short term horizons ($h = 1, 3$).

In Figure 5, the copula parameters are estimated with the pairs that give the highest correlation based on Kendall's τ. The estimated γ is larger for all forecasting horizons compared with the results where the copula function is fitted by the same horizons. $\hat{\gamma}$ increases as the horizon increases until $h = 10$. The strength of dependence of inflation uncertainty weakens until it reaches the local minimum at $h = 13$. The maximum value of the estimated γ occurs at $h = 20$. The rank correlation coefficients show similar patterns to the estimated copula parameter.

To showcase how the probability model by copulas can be interpreted in a similar fashion as in the 'at risk' approach in the literature, the conditional probability of the UK inflation being in a certain range conditional on the euro inflation uncertainty is illustrated in Appendix B.

IV Endogenous model of interdependence and measuring interdependence through heteroskedasticity

A Endogenous model of interdependence

The probability model to measure interdependence of inflation uncertainty could suffer from bias if endogeneity is not properly addressed. To illustrate the potential endogeneity bias in conventional empirical methods of measuring interdependence, an endogenous model of interdependence is assumed as in Rigobon (2019).

$$U_{1t} = \alpha U_{2t} + \eta_t \tag{15}$$

$$U_{2t} = \beta U_{1t} + \varepsilon_t \tag{16}$$

where U_{1t} is inflation uncertainty of the UK, U_{2t} is inflation uncertainty of euro area. The error terms, η_t and ε_t are assumed to be structural shocks, independent and have a Normal distribution. α and β are the coefficients capturing interdependence of uncertainty between two region. The variances of error terms are defined as σ_η^2 and σ_ε^2.

The reduced form equations of each country's uncertainty are as follows.

$$U_{1t} = \frac{1}{(1-\alpha\beta)}(\eta_t + \alpha\varepsilon_t) \tag{17}$$

$$U_{2t} = \frac{1}{(1-\alpha\beta)}(\beta\eta_t + \varepsilon_t) \tag{18}$$

The joint residuals in each of the above equations can be graphically represented by a rotated ellipse and the rotation can be summarized by the variance-covariance matrices.

$$\Omega = \begin{bmatrix} V_1 & C_{12} \\ C_{12} & V_2 \end{bmatrix} = \frac{1}{(1-\alpha\beta)^2} \begin{bmatrix} \alpha^2\sigma_\varepsilon^2 + \sigma_\eta^2 & \alpha\sigma_\varepsilon^2 + \beta\sigma_\eta^2 \\ \alpha\sigma_\varepsilon^2 + \beta\sigma_\eta^2 & \sigma_\varepsilon^2 + \beta^2\sigma_\eta^2 \end{bmatrix} \tag{19}$$

where $V_1 = var(U_{1t})$, $V_2 = var(U_{2t})$, $C_{12} = cov(U_{1t}, U_{2t})$. The parameters representing the underlying information of the system are the slopes of structural equations (α, β) and the relative volatility of the structural shocks, ($\theta \equiv \sigma_\eta^2/\sigma_\varepsilon^2$). Therefore, if estimated by conventional methods, such as correlation or principal components of two series, the coefficients of interdependence may be biased. For instance, as illustrated in the endogenous model, linear regression of U_{1t} and U_{2t} results in simultaneity bias. The interdependence measured by probabilistic models using copula also suffers from endogeneity, even though the estimation procedure does not assume any econometric models. The conditional probability of a tail-event can be driven by the changes either in σ_ε^2 or σ_η^2 or any combination of the two, and thereby, cannot distinguish changes in interdependence from those in heteroskedasticity.[16]

[16] See Appendix C for quantile regressions to provide some empirical evidence of endogeneity bias.

B Measuring Interdependence by Identification through Heteroskedasticity

To identify the system of equations, we exploit the heteroskedasticity in the data across different regimes, proposed by Rigobon (2003), Rigobon and Sack (2004), and Rigobon (2019).[17]

Rewriting the endogenous model of equations (19)-(20) in a matrix representation with structural shocks as follows.

$$A \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} \eta \\ \varepsilon \end{bmatrix} \tag{20}$$

where

$$A = \begin{bmatrix} 1 & -\alpha \\ -\beta & 1 \end{bmatrix} \tag{21}$$

Define Ω to be variance-covariance matrix of $[U_1 U_2]'$.

$$\Omega = \begin{bmatrix} V_1 & C_{12} \\ C_{12} & V_2 \end{bmatrix} \tag{22}$$

Then, the variance-covariance matrix of the structural model can be expressed as follows:

$$A\Omega A^T = \begin{bmatrix} 1 & -\alpha \\ -\beta & 1 \end{bmatrix} \begin{bmatrix} V_1 & C_{12} \\ C_{12} & V_2 \end{bmatrix} \begin{bmatrix} 1 & -\beta \\ -\alpha & 1 \end{bmatrix} \tag{23}$$

$$= \begin{bmatrix} \cdot & -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) \\ -\beta V_1 - \alpha V_2 + C_{12}(1+\alpha\beta) & \cdot \end{bmatrix} \tag{24}$$

By the assumption that η and ε are independent, the variance-covariance matrix of the structural model satisfies the following.

$$A\Omega A^T = \Sigma_{\eta\varepsilon} = \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{bmatrix} \tag{25}$$

The off-diagonal terms in Equation (30) need to be equal to zero, which defines the objective

[17] Recent applications includes Ehrmann et al. (2011) and Nakamura and Steinsson (2018).

function of the optimization problem.

$$\min_{\alpha,\beta} f(\alpha, \beta; V_1, V_2, C_{12}) = -\beta V_1 - \alpha V_2 + C_{12}(1 + \alpha\beta) \tag{26}$$

The estimation strategy using the identification through heteroskedasticity is to solve the system for two unknowns, α, β. In order to solve the problem, we impose additional assumptions that the parameter values in matrix A are stable over time and that the data have heteroskedasticity. We define two regimes that are determined by the ratio of variances, θ: RH if $\theta > median(\theta)$ and RL otherwise. The ratio (θ) of the variance of U_1 to the variance of U_2 is computed using different sample periods (p), forecast horizons (h), and rolling windows (rw).

$$\theta = \frac{V_1}{V_2} = m(p; h, rw) \tag{27}$$

where $p \in \{1, 2, 3\}$ while defining 1 is pre-crisis period, 2 Global Financial Crisis period, 3 post-crisis period. $h = 1, 2, \ldots, 24$ and $rw = 12$.[18] Then V_1, V_2, and C_{12} for each regime are computed to obtain minimum distance estimates of α and β.[19] The estimation model is just identified as we have two equations with two unknowns.

The significance of the estimates is computed by bootstrap. We bootstrap every regime, but not across the regimes, so that only the observations within each regime with replacement. Bootstrap is drawn from uniform distribution for 500 times.

Figure 6 presents the point estimates of the parameters, α and β, in Equation (19)-(20) against the forecast horizons ($h = 1, \cdots, 24$). The results from the pre-crisis period (top panel) show that both α and β are estimated to be closer to zero for $h < 12$, suggesting the interdependence of near term inflation uncertainty is statistically insignificant. For the longer horizons, the signs of coefficients alternate within the range of [-1, 1]. The pattern of opposite signs between α and β is also shown for $h > 12$.

[18] Pre-crisis periods: September 2003 - December 2007, Crisis period: January 2008 - December 2012, Post-crisis period: January 2013 - March 2016. We check robustness by changing p and rw.

[19] We allow positive and negative values for α and β by imposing sign restrictions $-1 \leq \alpha \leq 5, -1 \leq \beta \leq 5$.

The results for the crisis period (middle panel) show that the estimates of β from the crisis period exceed 1, in particular for longer term horizons. This could indicate potential amplifying effects of the surprises in the UK inflation on the euro area inflation during the GFC period. However, the crisis period estimates of α remain near zero for longer term forecast horizons, implying that the spillover of inflation uncertainty from the euro area to the UK is almost insignificant. For the near-term uncertainty (3-9 months ahead), the estimates of interdependence range between -1 and 1 and the signs of dependence measures are in the opposite direction: $\alpha > 0$ and $\beta < 0$.

For the post-crisis period (bottom panel), the range of the estimates lies between -1 and 1, mostly close to zero. For $h > 15$, β is estimated to be negative, implying that the upside (downside) inflation surprises in the UK are translated into the downside (upside) inflation uncertainty in the euro area. A negative sign of the estimated structural parameters may suggest that the underlying drivers of inflation uncertainty shocks do not stem from common factors such as high volatility in commodity prices affecting both economies in a same way.

Tables 6-8 show the point estimates of α or β as well as bootstrap results. At the 1 percent significance level, only the crisis-period estimates of β for $h = 11, 12, 20, 21, 22, 24$ are statistically significant and positive. At the 5 percent significance level, the crisis-period estimates of α for $h = 9, 10$ are significantly positive but less than 1, suggesting that the upside (downside) inflation uncertainty of the euro area is translated into the upside (downside) uncertainty of the UK but with a lesser extent.

V Conclusions

This paper explores various empirical methodologies to measure the strength of the interdependence of inflation uncertainty between the euro area and the UK. We first estimate inflation uncertainty by *ex post* forecast errors from a bivariate VAR GARCH model. It is shown that the estimated uncertainty is well characterised by non-Gaussian density with skewed, heavy tail properties–Two Piece Normal (TPN) and Weighted Skewed Normal (WSN) and the goodness-of-fit tests support the choice of WSN against TPN for both the UK and the euro area inflation uncertainty.

The estimated parameters in WSN suggests that the UK monetary policy reactions in the short run have been relatively hawkish while in the longer term the responses are rather dovish, focusing more on economic growth. For the euro area, the estimation results suggest that the ECB tends to be hawkish in response of upside risk of inflation uncertainty regardless of the forecast horizons.

The interdependence of uncertainty is estimated using a probability model. The results imply that the simultaneous spillover of inflation uncertainty is stronger for uncertainty about distant future than near future.

However, the evidence from quantile regressions indicates that such empirical method could suffer from endogeneity biases. To identify structural parameters in an endogeneity representation of interdependence, we exploit heteroskedasticity in the data across different regimes: pre-crisis, GFS crisis, and post-crisis periods. The results no longer exhibit stronger interdependence at longer horizons. The strength of the propagation of inflation uncertainty intensifies during the GFS period while the interdependence significantly dampens during the post-crisis period.

The main policy implications of this research is the importance of the monetary policy credibility in the presence of increased interdependence of inflation uncertainty during the crises periods. Heightened inflation uncertainty of a country may amplify uncertainty of the other country with strong economic and financial linkages, resulting in de-anchoring of inflation expectation. Monetary policy credibility becomes more crucial in times of great uncertainty with high degrees of contagion.

There are a number of gaps in the research that would benefit from further study. It includes the extension of the sample through Brexit, a crucial moments for the UK and EU relationships, as well as through the COVID-19 pandemic, a time where inflation uncertainty is among the highest over history. In addition, it would be helpful to understand the potential multiple relationships including the US.

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Tables and Figures

FIGURE 1 Inflation uncertainty

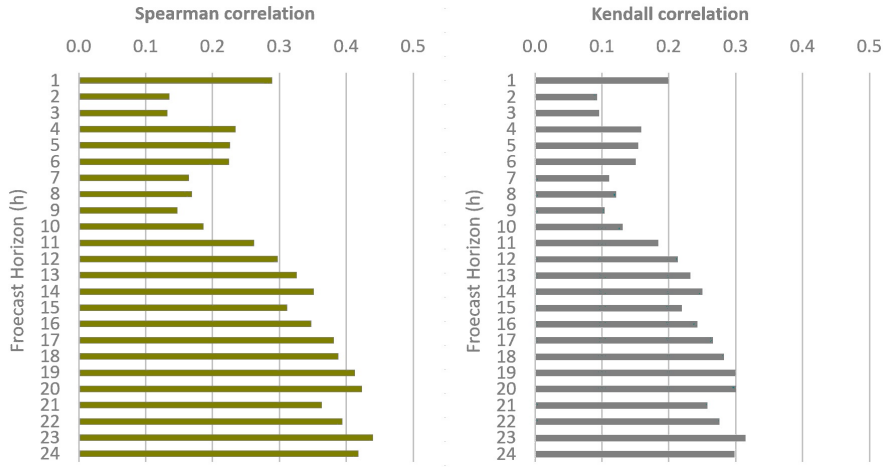
Notes: The figures are the estimated coefficients in Equation (1) by VAR BEKK GARCH (1,1) model.The top panel shows estimated inflation uncertainty of the UK for the selected forecast horizons, $h = 3, 6, 12, 18, 24$. The bottom panel shows estimated inflation uncertianty of the euro area for the same selected forecast horizons.

TABLE 1 Summary statistics of inflation uncertainty

	UK				Euro			
Horizon	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
1	-0.05	0.43	-0.19	0.35	-0.02	0.41	-0.25	0.42
2	-0.09	0.69	-0.06	0.61	-0.10	0.52	-0.01	0.41
3	-0.06	1.00	-0.13	1.12	-0.15	0.69	-0.24	1.82
4	-0.02	1.20	-0.08	1.24	-0.16	0.90	-0.81	2.95
5	-0.06	1.43	-0.06	1.21	-0.28	1.23	-1.10	4.06
6	-0.13	1.58	-0.02	1.63	-0.26	1.51	-1.13	4.66
7	-0.13	1.75	-0.01	2.22	-0.33	1.81	-1.23	4.99
8	-0.09	1.89	0.09	2.58	-0.45	2.14	-1.17	4.75
9	-0.10	1.97	0.25	3.53	-0.51	2.45	-1.15	5.13
10	-0.13	1.99	0.10	2.84	-0.59	2.68	-1.22	5.14
11	-0.16	2.03	0.09	2.86	-0.63	2.82	-1.32	5.33
12	-0.09	2.00	0.29	3.48	-0.51	2.95	-1.10	4.98
13	-0.14	1.93	0.23	2.81	-0.59	2.97	-1.07	4.66
14	-0.13	1.81	0.45	2.88	-0.73	2.98	-0.96	3.72
15	-0.09	1.70	0.48	3.23	-0.80	2.88	-0.78	3.50
16	-0.06	1.57	0.39	1.77	-0.81	2.76	-0.72	2.79
17	-0.08	1.45	0.40	1.12	-0.90	2.67	-0.59	1.94
18	-0.10	1.28	0.36	0.00	-0.83	2.54	-0.35	1.44
19	-0.08	1.21	0.43	-0.20	-0.85	2.45	-0.12	1.29
20	-0.03	1.24	0.65	0.84	-0.89	2.46	0.24	1.59
21	-0.03	1.24	0.65	1.15	-0.90	2.45	0.30	1.83
22	-0.02	1.33	0.94	2.19	-0.92	2.52	0.51	2.21
23	-0.03	1.37	0.95	1.75	-0.91	2.51	0.57	1.67
24	0.03	1.44	1.19	2.97	-0.76	2.47	0.60	1.63

Notes: Inflation uncertainty is measured by *ex post* forecast errors from a bivariate VAR BEKK GARCH using monthly inflation (January 1997-March 2016). To standardize, the forecast errors are multiplied by the square root of the unconditional variance-covariance matrix and divided by the square root of the conditional variance-covariance matrix of error terms.

FIGURE 2 Interdependece of inflation uncertainty: rank correlation coefficients



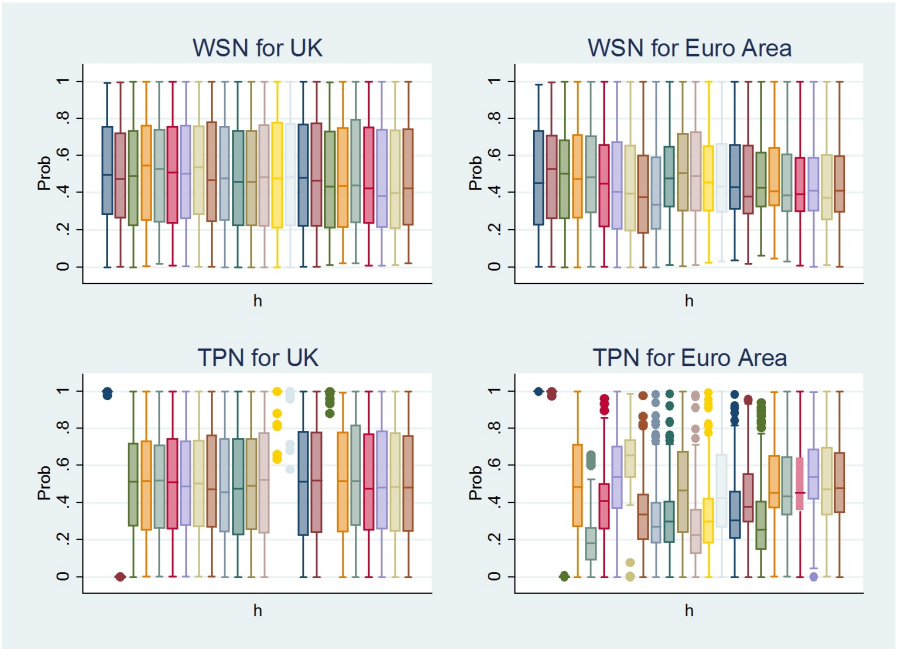
Notes: Spearman's rank correlation can be defined as $\rho_S(X,Y) = \rho(F_1(X), F_2(Y))$. Kendall's rank correlation is defined as $\rho_\tau(X,Y) = \Pr[(X_1 - X_2)(Y_1 - Y_2) > 0] - \Pr[(X_1 - X_2)(Y_1 - Y_2) < 0]$.

TABLE 2 The estimated parameters of marginal densities

		h=6		h=12		h=18		h=24	
		UK	Euro	UK	Euro	UK	Euro	UK	Euro
WSN	α	-1.81	-3.61	-1.47	-3.19	-0.84	-3.19	-1.00	-0.96
		(0.36)	(1.29)	(0.42)	(0.54)	(0.90)	(0.49)	(0.37)	(0.01)
	β	-0.98	-2.72	-1.38	-0.21	-0.95	0.00	-1.10	-0.01
		(0.46)	(1.01)	(0.69)	(0.17)	(1.54)	(0.00)	(1.09)	(0.02)
	σ	0.99	0.56	1.22	1.47	1.13	1.74	1.07	2.28
		(0.08)	(0.29)	(0.29)	(0.08)	(0.51)	(0.10)	(0.31)	(0.12)
	MD	**1.99**	**13.56**	**14.13**	**22.70**	12.64	**46.18**	6.65	24.86
TPN	σ_1	1.70	0.56	1.58	1.61	3.91	1.07	0.54	1.84
		(0.71)	(0.27)	(0.42)	(1.51)	(0.27)	(0.31)	(0.67)	(0.75)
	σ_2	1.03	2.76	1.78	1.83	0.23	3.99	1.78	2.60
		(0.21)	(1.12)	(0.47)	(0.71)	(0.20)	(0.03)	(0.03)	(0.89)
	μ	0.35	-1.12	-0.26	-0.32	-2.59	-1.18	-0.96	-1.38
		(0.40)	(2.02)	(0.70)	(1.51)	(0.62)	(0.83)	(0.51)	(1.20)
	MD	4.64	39.19	15.37	39.97	**6.56**	55.71	**3.05**	**18.38**
Sample size		146		140		134		128	

Notes: MD denotes the minimum distance statistics for the equiprobable null hypothesis against the alternative hypothesis of bumps or dips in the probability. Under the null hypothesis, the MD statistic has an asymptotic χ^2 distribution (Cressie and Read, 1984).

FIGURE 3 Box plot of probability integral transformation



Notes: h is horizon of uncertainty index, ranging from 1 to 24. The boxes of each plot indicate IQR (interquantile range) with median. The whiskers are stretched in both sides to 1.5 IQR and the outliers are presented in dots.

TABLE 3 Cramér-von Mises statistics for testing uniformity of *pit's*

	h=6		h=12		h=18		h=24	
	UK	Euro	UK	Euro	UK	Euro	UK	Euro
WSN	0.173	0.134	0.194	0.139	0.212	0.140	0.224	0.147
TPN	0.198	0.127	0.223	0.158	0.331	0.177	0.278	0.166

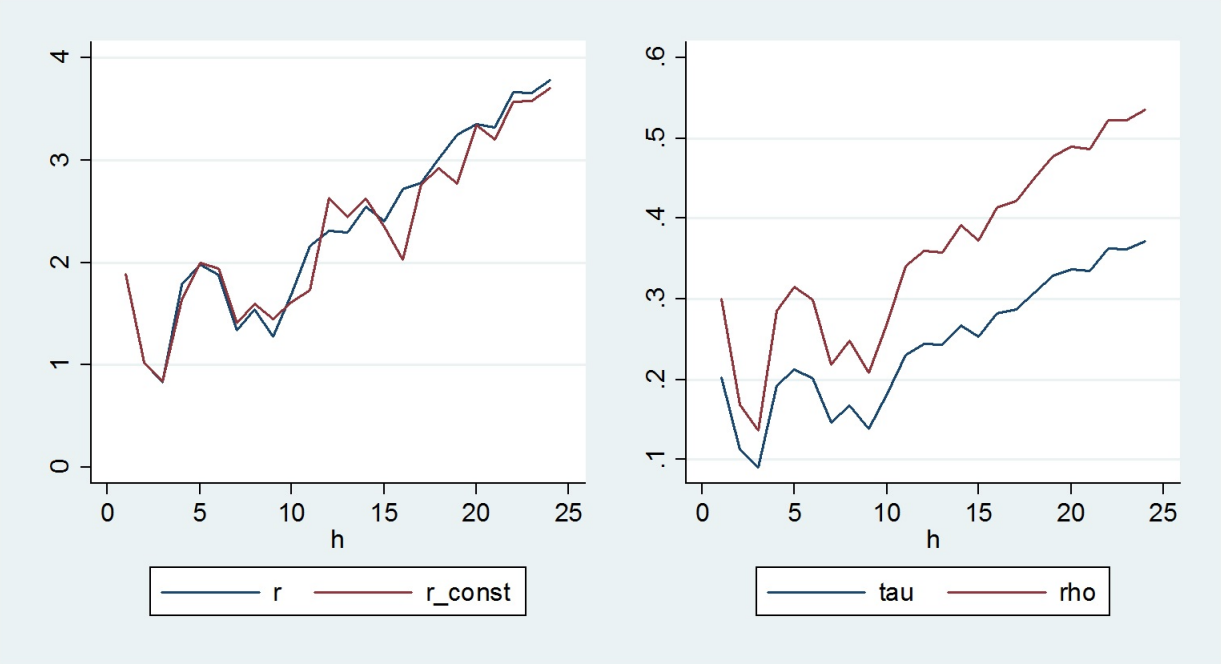
Notes: Asymptotic critical values for the Cramér-von Mises statistics are 0.347 at 10% significance level, 0.461 at 5% significance level.

TABLE 4 The estimated parameters of Frank copula: same horizon

h	γ	se(γ)	CvM	τ	ρ
1	1.886	0.783	0.091	0.203	0.300
2	1.022	0.784	0.094	0.112	0.168
3	0.823	0.751	0.083	0.091	0.136
4	1.783	0.800	0.120	0.192	0.285
5	1.984	0.837	0.219	0.212	0.315
6	1.878	0.830	0.193	0.202	0.299
7	1.341	0.820	0.151	0.146	0.218
8	1.539	0.848	0.201	0.167	0.249
9	1.273	0.830	0.332	0.139	0.208
10	1.684	0.858	0.425	0.182	0.271
11	2.167	0.874	0.311	0.230	0.340
12	2.307	0.876	0.202	0.244	0.360
13	2.297	0.838	0.194	0.243	0.358
14	2.552	0.849	0.210	0.267	0.392
15	2.401	0.833	0.253	0.253	0.372
16	2.713	0.841	0.286	0.282	0.413
17	2.777	0.819	0.333	0.287	0.421
18	3.017	0.856	0.395	0.309	0.451
19	3.252	0.860	0.374	0.329	0.478
20	3.354	0.868	0.460	0.337	0.490
21	3.326	0.873	0.354	0.335	0.487
22	3.665	0.893	0.419	0.362	0.523
23	3.654	0.872	0.463	0.362	0.522
24	3.784	0.913	0.256	0.372	0.536

Notes: The parameters are estimated under the assumption that ρ in WSN distribution decays exponentially as forecast horizon increases.

FIGURE 4 The estimated copula parameters and rank correlations: same horizon



Notes: r denotes γ parameter of Frank copula estimated under the assumption of decaying ρ in WSN marginal densities. *r_const* denotes γ parameter of Frank copula assuming constant ρ (=0.75) for WSN marginal densities. *tau* and *rho* are the estimated rank correlation coefficients computed by the analytical form (Equations 32 and 33) in Appendix A.

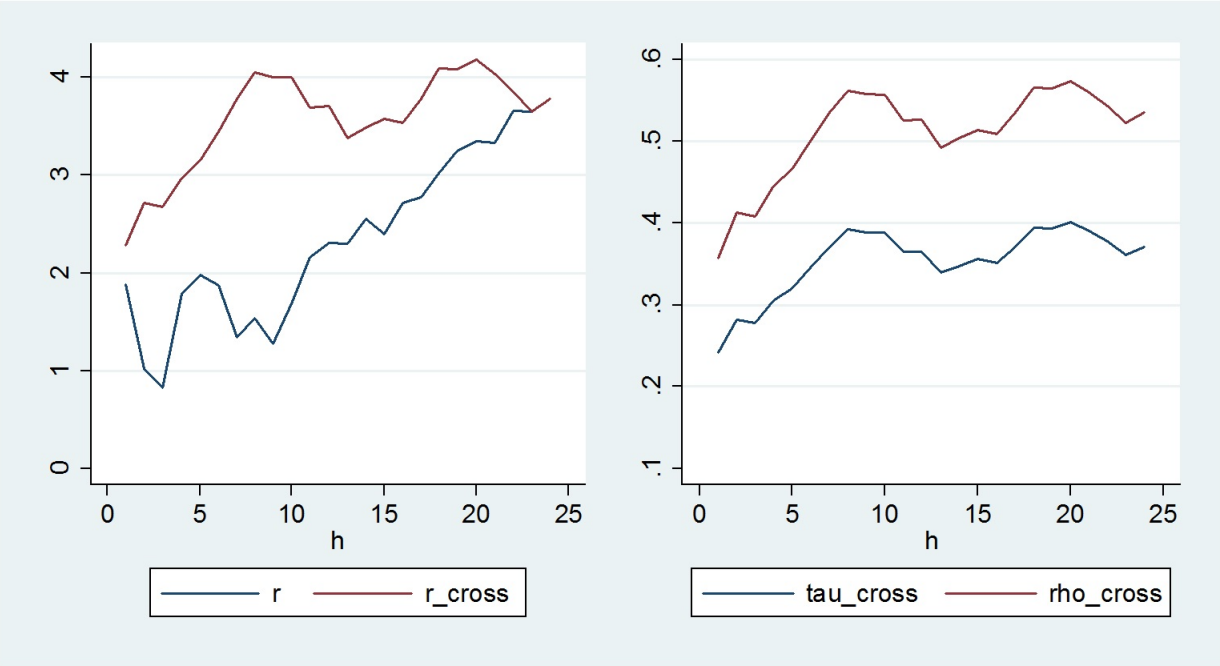
TABLE 5 The estimated rank correlations: matching horizons

h_{uk}	$h_{eu,1}$	τ	$h_{eu,2}$	ρ
1	12	0.223	6	0.316
2	12	0.261	12	0.358
3	12	0.259	8	0.369
4	13	0.268	13	0.378
5	14	0.273	14	0.384
6	14	0.303	14	0.418
7	14	0.322	14	0.450
8	14	0.326	14	0.448
9	14	0.321	14	0.435
10	14	0.306	16	0.430
11	14	0.307	17	0.435
12	17	0.299	17	0.424

h_{uk}	$h_{eu,1}$	τ	$h_{eu,2}$	ρ
13	19	(0.307)	20	(0.436)
14	20	(0.324)	20	(0.457)
15	20	(0.331)	20	(0.474)
16	23	(0.330)	23	(0.472)
17	23	(0.349)	23	(0.492)
18	23	(0.364)	23	(0.520)
19	23	(0.370)	24	(0.515)
20	24	(0.356)	24	(0.492)
21	24	(0.328)	24	(0.457)
22	24	(0.311)	24	(0.432)
23	23	(0.315)	23	(0.440)
24	24	(0.298)	24	(0.418)

Note: h_{uk} denotes the horizons for the UK inflation uncertainty. $h_{eu,1}$ denotes the horizons of the euro inflation uncertainty that give the highest Kendall's τ correlation given each horizon of the UK inflation uncertainty (h_{uk}). $h_{eu,2}$ refers to the horizons of the euro inflation uncertainty that give the highest Spearman's ρ correlation given each horizon of the UK inflation uncertainty (h_{uk}).

FIGURE 5 The estimated copula parameters and rank correlations: matching horizons



Notes: Given each forecast horizon of the UK inflation uncertainty, the forecast horizons of the euro area inflation uncertainty are selected to have the highest correlation. *r* denotes γ parameter of Frank copula estimated under the assumption of decaying ρ in WSN marginal densities and combining the same horizon. *r_cross* denotes γ parameter of Frank copula assuming decaying ρ in WSN marginal densities and combining different horizons that give the highest correlation. *tau_cross* and *rho_cross* are the estimated rank correlation coefficients that correspond to r_cross computed by the analytical form (Equations 32 and 33) in Appendix A.

FIGURE 6 Interdependece of inflation uncertainty: identification through heteroskedasticity

Notes: The figure shows the point estimates of α and β computed from Equation (27) using pre-crisis, crisis, and post-crisis periods, respectively. UK denotes the estimates of α in Equation (15) and EU denotes the estimated of β in Equation (16).

TABLE 6 The parameter estimates and bootstrap results: pre-crisis period

	α				β			
	Point estimate	Bootstrap			Point estimate	Bootstrap		
		Mean	SD	Prob ($\alpha < 0$)		Mean	SD	Prob ($\beta < 0$)
h=1	0.278	0.253	0.253	0.124	0.077	0.075	0.350	0.309
h=2	0.171	0.106	0.510	0.401	0.075	0.092	0.356	0.363
h=3	0.301	0.141	0.543	0.381	-0.105	-0.003	0.338	0.537
h=4	-0.058	0.022	0.571	0.495	0.051	-0.004	0.326	0.471
h=5	0.290	0.235	0.489	0.240	-0.187	-0.160	0.287	0.766
h=6	0.271	0.230	0.541	0.337	-0.297	-0.261	0.252	0.914*
h=7	0.010	0.012	0.414	0.527	-0.300	-0.288	0.201	0.950**
h=8	-0.143	-0.098	0.357	0.645	-0.292	-0.303	0.198	0.954**
h=9	-0.338	-0.235	0.473	0.739	-0.224	-0.239	0.375	0.743
h=10	-0.131	-0.236	0.614	0.671	-0.295	-0.142	0.435	0.593
h=11	-0.497	-0.162	0.614	0.629	0.035	-0.118	0.372	0.625
h=12	0.123	-0.077	0.603	0.555	-0.238	-0.059	0.441	0.563
h=13	0.555	0.023	0.556	0.401	-0.617	-0.131	0.517	0.637
h=14	0.732	-0.162	0.430	0.661	-0.984	0.103	0.566	0.425
h=15	0.165	0.161	0.300	0.234	-0.333	-0.296	0.372	0.788
h=16	0.242	0.209	0.200	0.106	-0.336	-0.310	0.281	0.844
h=17	0.076	0.049	0.491	0.439	-0.142	-0.101	0.344	0.605
h=18	-0.749	-0.568	0.462	0.908*	0.742	0.630	0.321	0.042++
h=19	-0.588	-0.529	0.321	0.958**	0.689	0.655	0.234	0.014++
h=20	-0.133	-0.016	0.315	0.719	0.170	0.153	0.133	0.136
h=21	0.107	0.312	0.471	0.305	-0.159	-0.206	0.241	0.844
h=22	0.450	0.444	0.398	0.034++	-0.417	-0.394	0.257	0.952**
h=23	0.117	0.102	0.255	0.307	-0.351	-0.350	0.324	0.932*
h=24	-0.215	-0.202	0.173	0.880	-0.310	-0.324	0.291	0.926*

Notes: The within-regime bootstrapped estimates are drawn from uniform distribution for 500 times. Then the probability of the estimated α, β to be below zero is computed for each regime and horizon.

TABLE 7 The parameter estimates and bootstrap results: crisis period

	α				β			
	Point estimate	Bootstrap			Point estimate	Bootstrap		
		Mean	SD	Prob ($\alpha < 0$)		Mean	SD	Prob ($\beta < 0$)
h=1	-0.539	-0.103	0.673	0.605	0.502	0.211	0.459	0.275
h=2	0.002	-0.005	0.536	0.501	0.196	0.160	0.207	0.178
h=3	0.825	0.350	0.737	0.277	-0.093	0.080	0.370	0.441
h=4	0.898	0.560	0.749	0.186	-0.269	0.015	0.510	0.607
h=5	0.783	0.613	0.653	0.136	-0.425	-0.136	0.703	0.750
h=6	0.705	0.564	0.543	0.134	-0.674	-0.301	0.735	0.808
h=7	0.657	0.532	0.482	0.132	-0.657	-0.240	0.853	0.770
h=8	0.616	0.541	0.536	0.118	-0.755	-0.360	0.791	0.818
h=9	0.315	0.599	0.538	0.044++	0.635	1.401	1.468	0.036++
h=10	0.268	0.468	0.417	0.034++	0.732	1.415	1.492	0.016++
h=11	0.238	0.385	0.380	0.082+	0.844	1.420	1.375	0.008+++
h=12	0.190	0.284	0.319	0.114	0.958	1.346	1.183	0.006+++
h=13	0.210	0.255	0.256	0.104	0.953	1.043	0.948	0.036++
h=14	0.172	0.170	0.151	0.110	1.044	1.019	0.428	0.012++
h=15	0.185	0.173	0.129	0.090+	0.989	0.928	0.428	0.032++
h=16	-0.020	0.030	0.178	0.507	1.473	1.208	0.715	0.082+
h=17	-0.012	0.039	0.168	0.457	1.552	1.243	0.773	0.096+
h=18	-0.140	0.056	0.237	0.585	1.921	0.963	1.282	0.293
h=19	-0.019	0.002	0.129	0.557	1.749	1.546	0.715	0.066+
h=20	0.042	0.040	0.057	0.214	1.608	1.600	0.175	0.000+++
h=21	0.065	0.062	0.049	0.100+	1.638	1.630	0.137	0.000+++
h=22	0.053	0.053	0.056	0.176	1.622	1.614	0.110	0.000+++
h=23	0.350	0.262	0.171	0.096+	-0.022	0.274	1.028	0.447
h=24	0.078	0.083	0.095	0.174	1.438	1.422	0.266	0.000+++

Notes: The within-regime bootstrapped estimates are drawn from uniform distribution for 500 times. Then the probability of the estimated α, β to be below zero is computed for each regime and horizon.

TABLE 8 The parameter estimates and bootstrap results: post-crisis period

	α				β			
	Point estimate	Bootstrap			Point estimate	Bootstrap		
		Mean	SD	Prob ($\alpha < 0$)		Mean	SD	Prob ($\beta < 0$)
h=1	0.079	0.060	0.353	0.477	0.307	0.251	0.652	0.309
h=2	-0.406	-0.273	0.340	0.838	0.737	0.493	0.643	0.196
h=3	-0.170	-0.135	0.376	0.661	0.144	0.082	0.519	0.363
h=4	-0.396	-0.240	0.398	0.838	0.707	0.489	0.576	0.146
h=5	-0.278	-0.218	0.405	0.826	-0.046	-0.050	0.216	0.607
h=6	0.597	0.165	0.654	0.349	-0.563	-0.221	0.516	0.747
h=7	-0.179	-0.150	0.323	0.705	-0.050	-0.055	0.316	0.563
h=8	-0.410	-0.367	0.326	0.916*	0.269	0.237	0.338	0.192
h=9	0.530	0.455	0.508	0.096+	-0.175	-0.143	0.367	0.733
h=10	0.516	0.458	0.374	0.064+	-0.287	-0.230	0.384	0.800
h=11	0.615	0.477	0.519	0.104	-0.306	-0.209	0.372	0.818
h=12	0.566	0.409	0.487	0.138	-0.240	-0.112	0.447	0.747
h=13	0.287	0.279	0.275	0.136	-0.012	-0.005	0.292	0.525
h=14	-0.089	-0.119	0.211	0.741	0.252	0.303	0.273	0.132
h=15	0.319	0.291	0.200	0.062+	-0.613	-0.569	0.317	0.974**
h=16	0.275	0.231	0.262	0.136	-0.525	-0.443	0.450	0.886
h=17	0.297	0.099	0.348	0.265	-0.728	-0.318	0.683	0.768
h=18	0.267	0.147	0.281	0.166	-0.850	-0.560	0.636	0.874
h=19	0.139	0.118	0.165	0.152	-0.637	-0.582	0.396	0.952**
h=20	0.138	0.151	0.157	0.102	-0.715	-0.694	0.295	0.986**
h=21	0.050	0.060	0.122	0.363	-0.515	-0.516	0.233	0.984**
h=22	-0.028	-0.022	0.119	0.603	-0.352	-0.333	0.274	0.918*
h=23	0.053	0.055	0.114	0.309	-0.467	-0.462	0.241	0.984**
h=24	-0.027	-0.055	0.215	0.593	-0.266	-0.104	0.688	0.685

Notes: The within-regime bootstrapped estimates are drawn from uniform distribution for 500 times. Then the probability of the estimated α, β to be below zero is computed for each regime and horizon.

Appendix

A Frank copula

The *cdf* of Frank copula is given by

$$C(y_1, y_2; \gamma) = -\frac{1}{\gamma} \ln \left(1 + \frac{(e^{-\gamma y_1} - 1)(e^{-\gamma y_2} - 1)}{e^{-\gamma} - 1} \right) \tag{28}$$

where $\gamma \in (-\infty, +\infty)$. If $\gamma = 0$, the copula is independent.

The copula generator for Frank copula, $\varphi(\cdot)$, is

$$\varphi_\gamma(t) = -\ln \left(\frac{e^{-\gamma t} - 1}{e^{-\gamma} - 1} \right) \tag{29}$$

The *pdf* of Frank copula is

$$c(y_1, y_2; \gamma) = \frac{-\gamma(e^{-\gamma} - 1)e^{-\gamma(y_1 + y_2)}}{((e^{-\gamma y_1} - 1)(e^{-\gamma y_2} - 1) + (e^{\gamma} - 1))^2} \tag{30}$$

The analytical closed forms of these rank correlations, which depend on the parameter value, γ, are as follows.

$$g_\tau(\gamma) = 1 - \frac{4(1 - D_1(\gamma))}{\gamma} \tag{31}$$

$$g_\rho(\gamma) = 1 - \frac{12(D_1(\gamma) - D_2(\gamma))}{\gamma} \tag{32}$$

where $D_k = kx^{-k} \int_0^x t^k (e^t - 1)^{-1} dt$ is the Debye function.

B Conditional probability

Based on the estimated marginal and joint densities of inflation uncertainty, the conditional probability of certain scenarios of inflation outcomes can be computed. The subscript for the density functions (f and F) and uncertainty index (U) indicates each region: 1 for the UK and 2 for the euro area. Then, the unconditional probability for the UK inflation being inside [a,b] can be represented as follows.

$$\int_a^b \hat{f}_1(U_1)\, dU_1 \tag{33}$$

The probability of the UK inflation inside the interval [a,b] conditional on the euro inflation

being inside the same interval is given below.

$$\frac{\int_a^b \int_a^b c(\hat{F}_1, \hat{F}_2; \hat{\alpha})\ \hat{f}_1 \cdot \hat{f}_2\ dU_1 dU_2}{\int_a^b \hat{f}_2(U_2)\ dU_2} \tag{34}$$

Table B1 and Figure B1 show two different scenarios, (i) the inflation below 1% and (ii) the inflation within [1%, 3%] for both economies based on the post-crisis sample period.

The downward sloping lines in the left panel of Figure B1 suggest that both unconditional and conditional probability of the UK inflation below 1% decreases as forecasting horizon increases. The unconditional probability is lower than the conditional probability for all horizons in the first scenario. For example, the probability of the UK inflation below 1% in two years without considering the dependence structure is approximately 0.24 while this increases to 0.31 if it is known that the euro area inflation will also become below 1% in two years. This implies that the left tail events of inflation are positively correlated between the two regions.

The second scenario of the UK inflation within the target band tells a different story (Figure B1 right panel). The probabilities do not decrease in a monotonic sense when forecast horizon increases. The unconditional probability appears to be flat across all forecast horizons, roughly between 0.4 and 0.5. The conditional probabilities are either flat (copula estimated with marginals of the same horizons) or increasing (copula estimated with marginals of the different horizons) for the short forecast horizons. For the longer horizons, the conditional probabilities tend to decline as the forecast horizon increases. Comparing the unconditional and conditional probabilities, unconditional probability of the UK inflation inside the target band is significantly lower than the conditional probability in the short and medium term. However, the long term unconditional probability is larger than the conditional probability.

TABLE B1 The unconditional and conditional probability of the UK inflation

I. The probability of the UK inflation below 1%

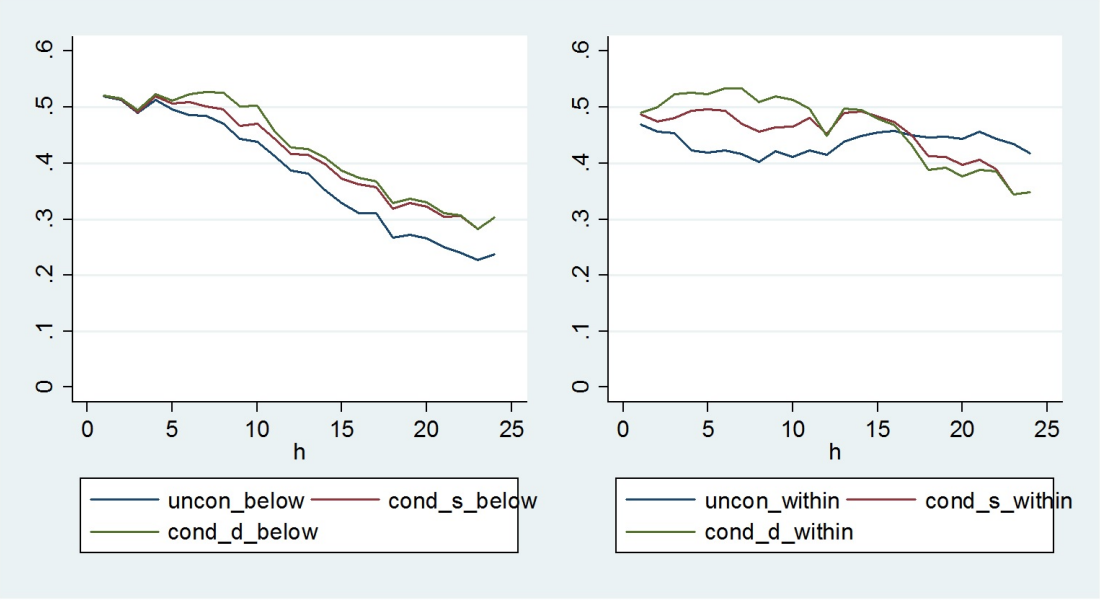
	Unconditional	Conditional (same h)	Conditional (different h)	$[h_{eu}]$
$h_{uk} = 6$	0.4867	0.5095	0.5238	14
$h_{uk} = 12$	0.3863	0.4163	0.4281	17
$h_{uk} = 18$	0.2663	0.3184	0.3289	23
$h_{uk} = 24$	0.2387	0.3036	0.3036	24

II. The probability of the UK inflation within [1%, 3%]

	Unconditional	Conditional (same h)	Conditional (different h)	$[h_{eu}]$
$h_{uk} = 6$	0.4224	0.4942	0.5337	14
$h_{uk} = 12$	0.4150	0.4523	0.4488	17
$h_{uk} = 18$	0.4457	0.4127	0.3884	23
$h_{uk} = 24$	0.4179	0.3483	0.3483	24

Notes: The conditional probability is computed from the copula estimation results in Section III. B. *Conditional (same h)* indicates the conditional probability calculated using the estimated joint distribution combined by the same horizon univariate densities of the UK and the euro inflation uncertainty. *Conditional (different h)* indicates the conditional probability calculated using the estimated joint distribution combined by the matching univariate densities of the UK and the euro inflation uncertainty which give the highest Kendall's τ rank correlation given the horizon of the UK inflation uncertainty (h_{uk}). h_{eu} denotes the selected horizons for the euro inflation uncertainty that gives the highest Kendall's τ correlation.

FIGURE B1 The unconditional and conditional probabilities of the UK inflation



Notes: The left panel shows the results when the UK inflation is below 1%, and the right panel shows the results when the UK inflation is within target [1%, 3%]. The blue lines are unconditional probability and the red lines are conditional probability computed for the same horizons. The green lines are conditional probability computed for the different horizons when when pairing the two marginal densities.

C Empirical evidence of endogeneity bias: Quantile regressions

Quantile regressions (28) can provide empirical evidence of endogeneity. The model for the τ-th conditional quantile of Y, Q_τ, can be written as follows.

$$Q_\tau = X\beta(\tau) \tag{35}$$

where X is a vector of explanatory variables. The estimation of $\beta(\tau)$ is based on a sample of n observations in Y. If the estimates of $\beta(\tau)$ differ across τ's, it suggests that the marginal effect of X is heterogeneous across different quantiles of the conditional distribution of Y. Therefore, non-flat quantile treatment effects indicate whether there is any potential non-linearity in the data conditional on the quantile.

To detect potential bias due to endogeneity, we estimate the coefficients of quantile regressions of U_{1t} on U_{2t}, and vice versa. Figure C1 plots the estimated slope coefficients, $\beta(\tau)$, of the quantile regressions of U_{1t} on U_{2t} for each forecast horizon ($h = 1, \cdots, 24$). Different patterns among different groups of forecasting horizons were found in the results. The estimates from the regressions of uncertainty at short horizons exhibit large variations, while the estimates are likely to be flatter for the regressions of uncertainty at longer horizons. For the inflation uncertainty at short horizons ($h = 1, \cdots, 4$), the quantile slope estimates are U-shaped, suggesting the changes in skewness. For longer horizons ($h = 5, \cdots, 10$), the quantile slopes are flat up to 40-60 percentile and then U-shaped in the right tail. The estimates from uncertainty about one-year-ahead to 19-month-ahead inflation exhibit modest increases, with some irregularities as reaching the far right tail. The quantile regression coefficients tend to be flat for $h = 20, \cdots, 24$.

Figure C2 plot the quantile estimates of U_{2t} on U_{1t} for each forecast horizon. The plots are quite different from the previous results in many aspects. First, the quantile slope coefficients are much larger than those from the regressions of the UK uncertainty on the euro uncertainty.[20] Second, heterogeneity in the strength of interdependence is more pronounced in the case of uncertainty at longer horizons compared to the previous results. The variability of the estimated coefficients across quantiles tends to be greater as h gets larger. Third, we observe pick-ups in the estimated coefficients in the left tails except for very short ($h = 1, 2$) or very long horizons ($h = 23, 24$).

Next, we conduct simulations of quantile regressions with iid errors to confirm whether the quantile slope coefficients are flat across τ's when there is no endogeneity. First, we assume a model of random variables X and Y: $Y = 0.5X + \epsilon$, where the relationship between X and Y is uniquely defined by an iid error term, exogenous to X. Second, we assume a random variable X and an error term ϵ have following distributions. For each combination, we allow for different symmetric and asymmetric distributions while maintaining iid assumptions for ϵ's.

[20] Please note that y-axis is modified, ranging from 0 to 2. For $h = 19$, y-axis ranges from 0 to 2.2.

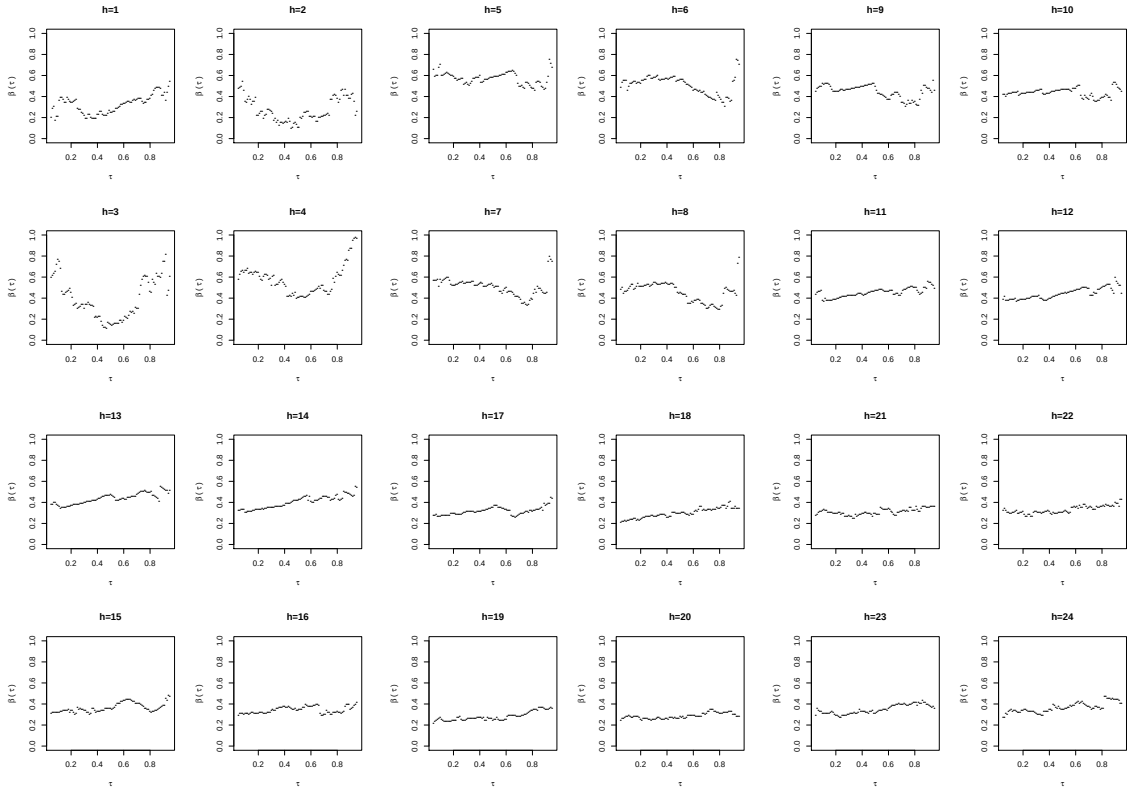
1. $X \sim N(2,1)$; $\epsilon \sim iidN(0,1)$.
2. $X \sim t(5)$; $\epsilon \sim iidN(0,1)$.
3. $X \sim t(5)$; $\epsilon \sim iidt(5)$.
4. $X \sim N(2,1) + \chi^2(4)$; $\epsilon \sim iidN(0,1)$.
5. $X \sim N(2,1)$; $\epsilon \sim iidt(5)$.
6. $X \sim N(2,1)$; $\epsilon \sim iidN(2,1) + \chi^2(4)$.

Third, we draw a sample of size $n = 1001$ for X, which is fixed over replications. Fourth, we draw error terms for $r = 100$ times and compute Y for each replication. Lastly, for every replication, we compute $\widehat{\beta(\tau)}$ and average $\widehat{\beta(\tau)}$ over replications.

Figure C3 present the estimated $\beta(\tau)$ from a sample draw (left panel) and from the average of all estimates across 100 repetitions (right panel). As expected, the slope coefficients are flat across quantiles when there is no endogeneity.

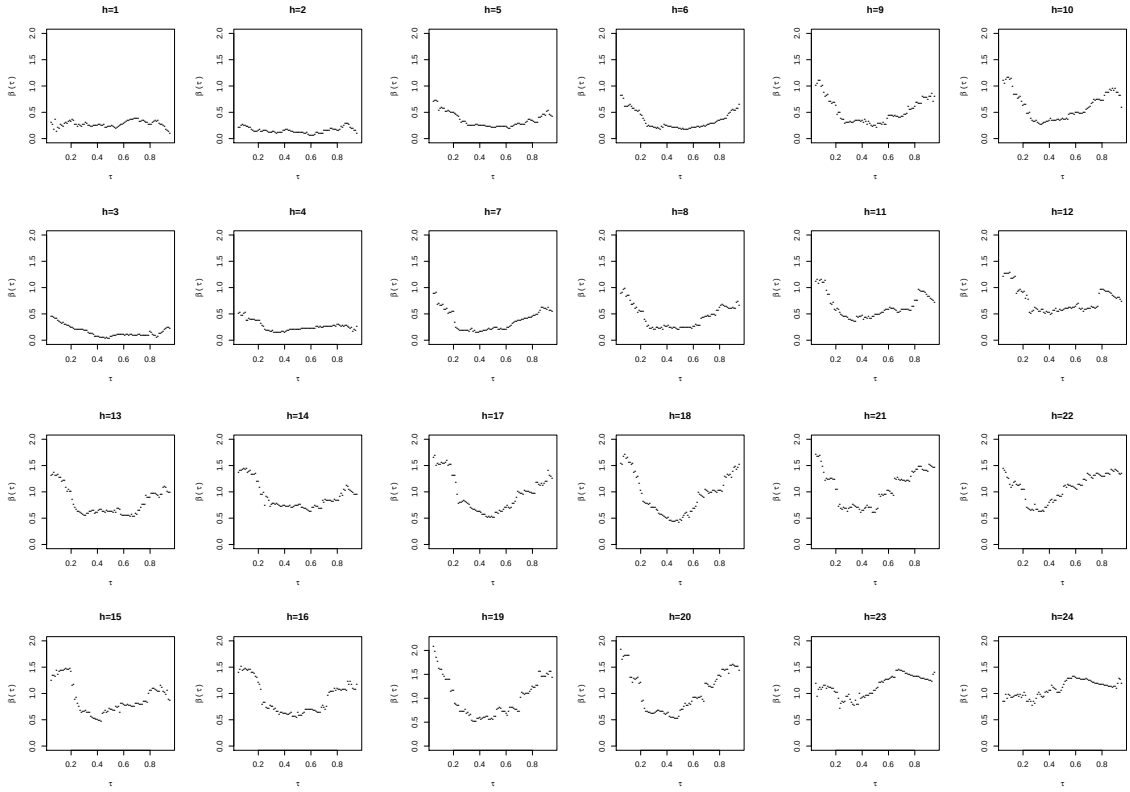
Overall, the results confirm that the conventional strategies for estimating interdependence could be biased in the presence of endogeneity problem.

Figure C1 The estimated slope coefficients of linear quantile regressions (I)



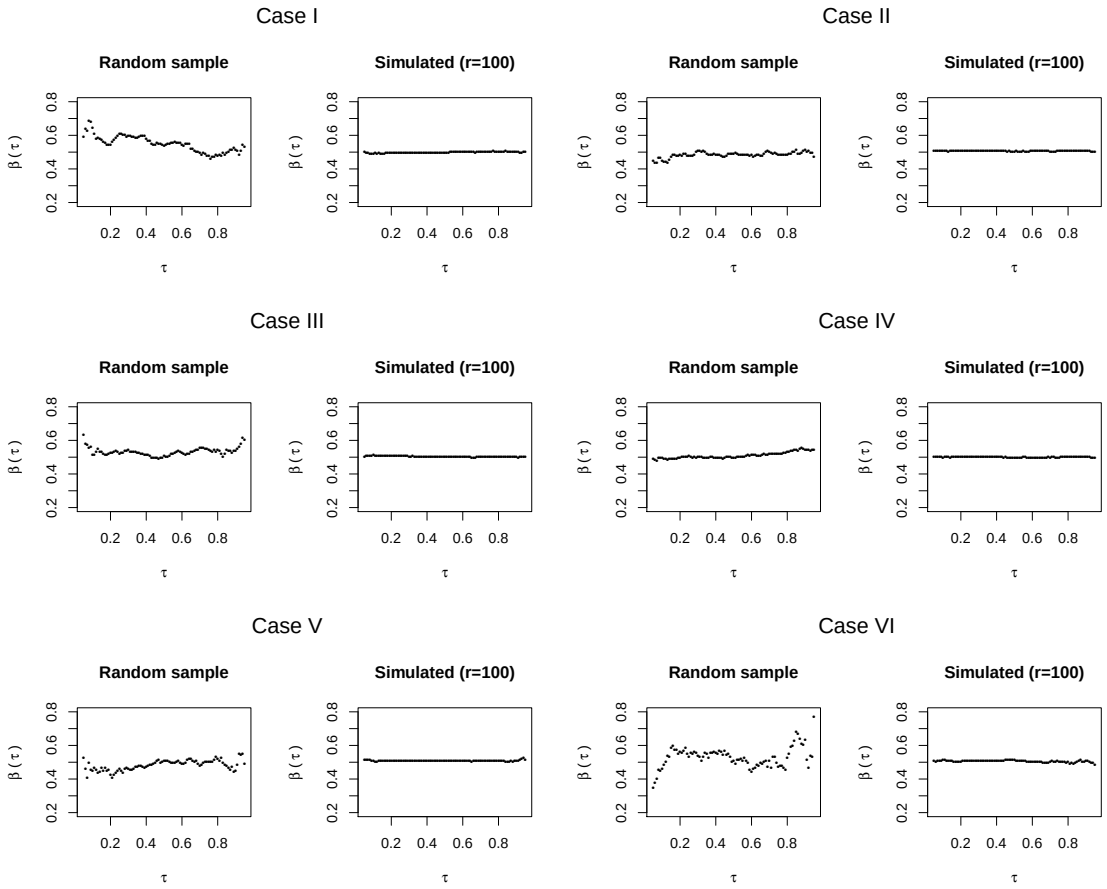
Notes: The estimation results from the quantile regressions of the UK inflation uncertainty on the euro inflation uncertainty for each horizon are presented. The estimated coefficients are plotted as a function of τ. The x-axis is $\tau = 0.05, 0.06, \cdots, 0.94, 0.95$. Y-axis is fixed across the figures for comparison.

Figure C2 The estimated slope coefficients of linear quantile regressions (II)



Notes: The estimation results from the quantile regressions of the euro inflation uncertainty on the UK inflation uncertainty for each horizon are presented. The estimated coefficients are plotted as a function of τ. The x-axis is $\tau = 0.05, 0.06, \cdots, 0.94, 0.95$. Y-axis is fixed across the figures for comparison.

Figure C3 The estimated slope coefficients of linear quantile regression (simulated)



Notes: The estimated coefficients, $\widehat{\beta(\tau)}$, using simulated data are plotted as a function of τ. A model of random variables X and Y: $Y = 0.5X + \varepsilon$. Case I: $\varepsilon \sim iidN(0,1)$, $X \sim N(2,1)$. Case II: $\varepsilon \sim iidN(0,1)$, $X \sim t(5)$. Case III: $\varepsilon \sim iid\, t(5)$, $X \sim t(5)$. Case IV: $\varepsilon \sim iidN(0,1)$, $X \sim N(2,1) + \chi^2(4)$. Case V: $\varepsilon \sim iid\, t(5)$, $X \sim N(2,1)$. Case VI: $\varepsilon \sim iidN(2,1) + \chi^2(4)$, $X \sim N(2,1)$. The sample size is $n = 1001$ for drawing X. The error terms are drawn for $r = 100$ times to compute Y.